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Multi-scale modeling of turbulent oceanic flows

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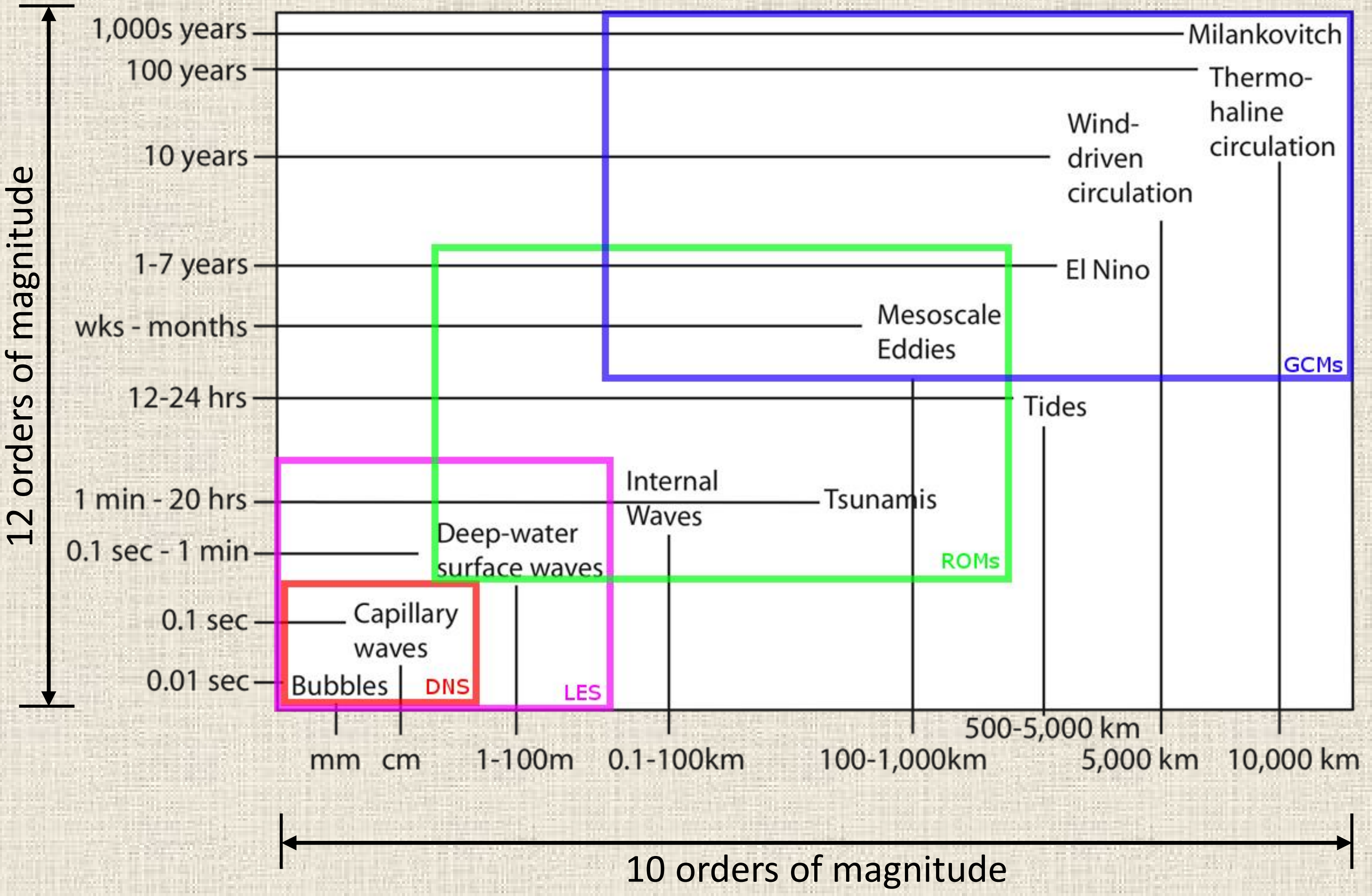
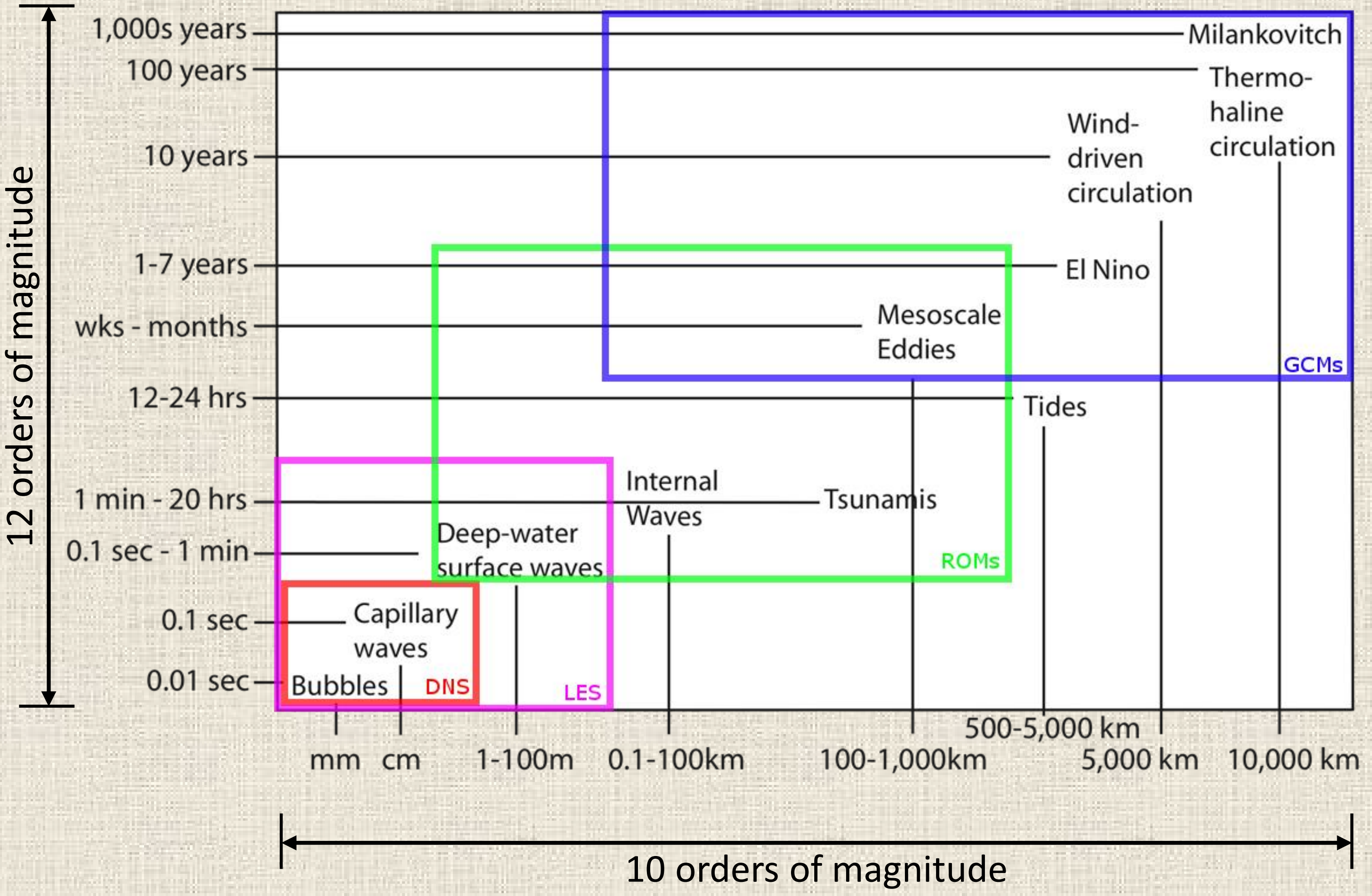
Modeling Challenges



- **LES** – Turbulence parameterized.
- **DNS** – Turbulent scales resolved.
- **Regional Ocean Models** – Captures nonlinear energy transfers. Requires a nonhydrostatic model.
- **Global Circulation Models** – Small scale effects poorly parameterized.

Image: http://pordlabs.ucsd.edu/ltalley/sio210/introduction/index.html

Modeling Challenges



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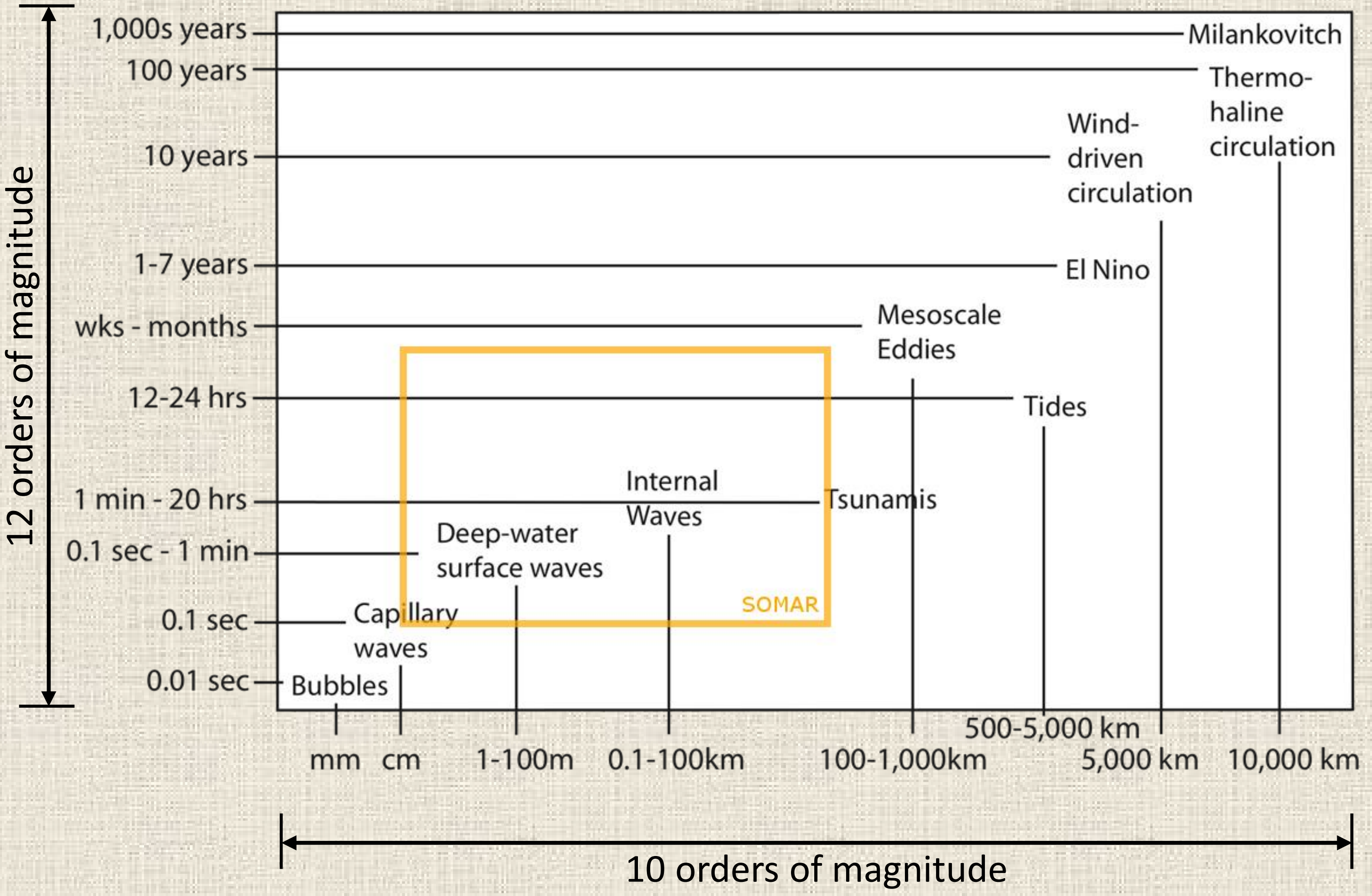
SOMAR

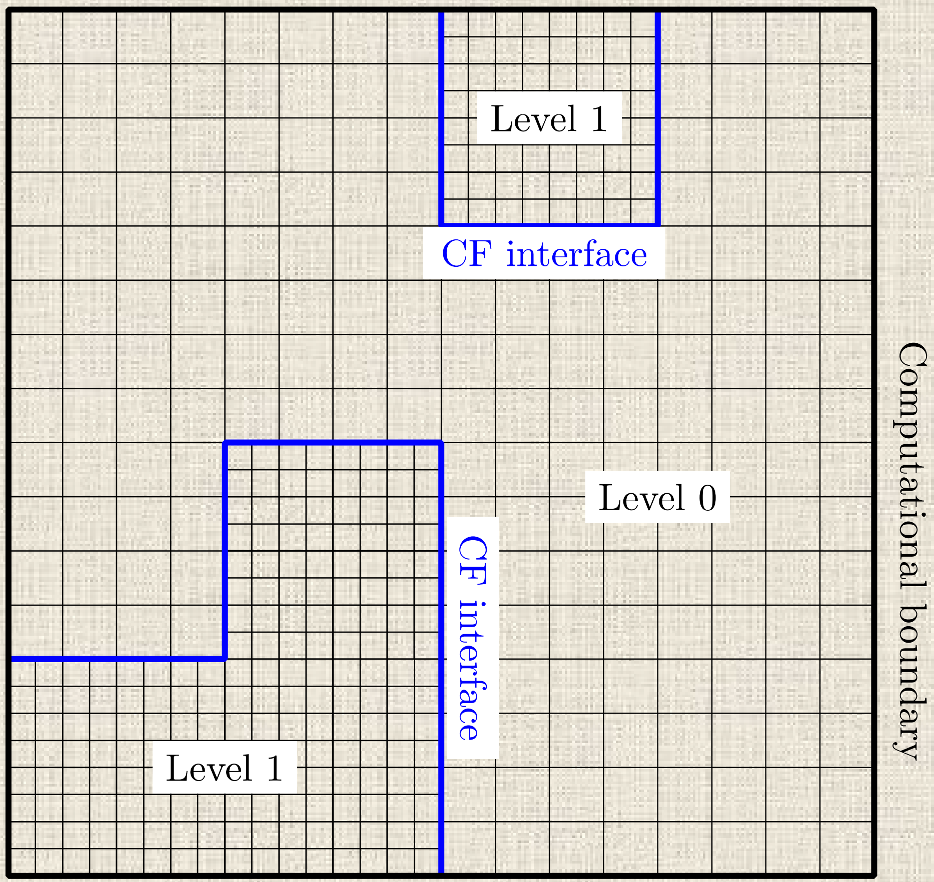
Stratified Ocean Model with Adaptive Refinement



- Non-hydrostatic 2D and 3D flows.
- Adaptive refinement in both space and time.
- Efficient anisotropic Poisson solvers.
- Non-orthogonal, curvilinear coordinates.
- Accepts very general subgrid stress info.
- Built on the Chombo framework.

SOMAR: Santilli & Scotti (*Journal of Computational Physics,* 2011 & 2015)
Chombo: https://commons.lbl.gov/display/chombo

SOMAR
Stratified Ocean Model with Adaptive Refinement



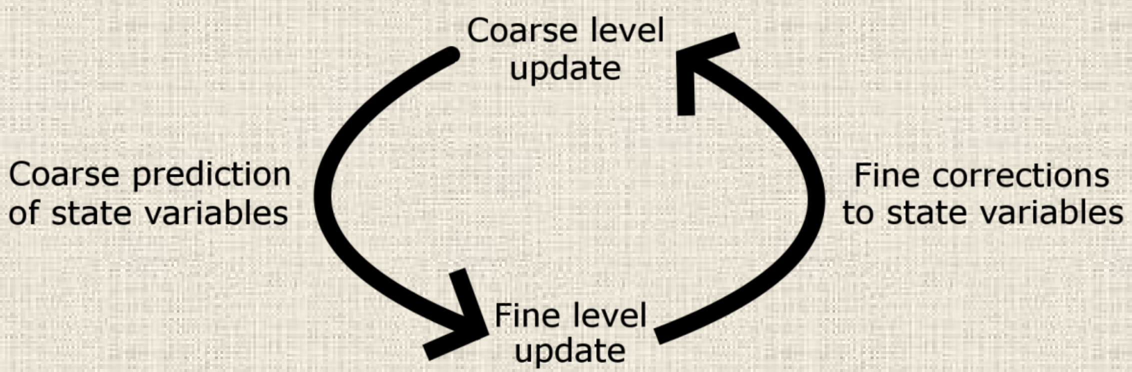
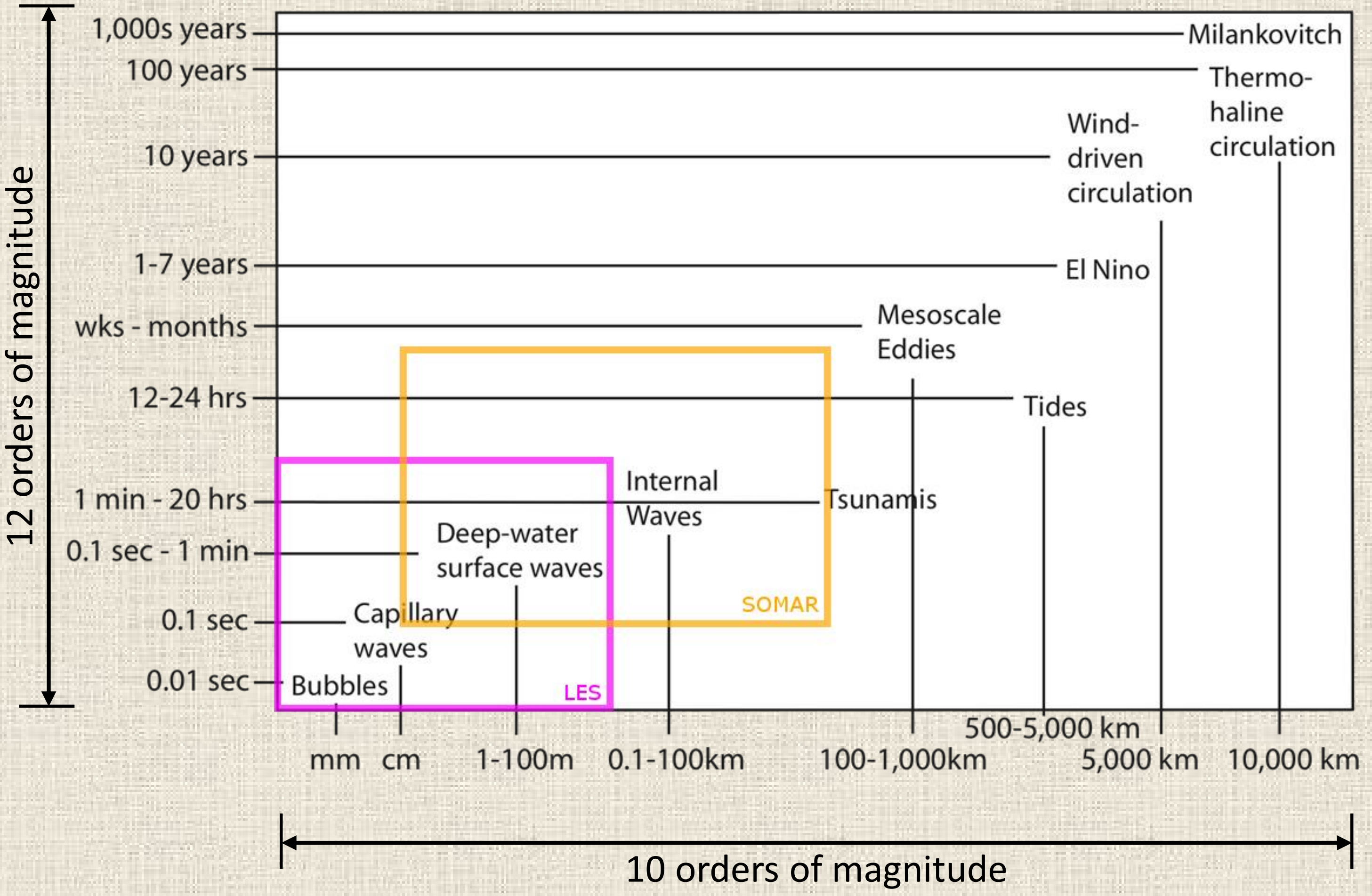


Image: Chalamalla et al. (*Ocean Modelling,* 2017)

SOMAR-LES



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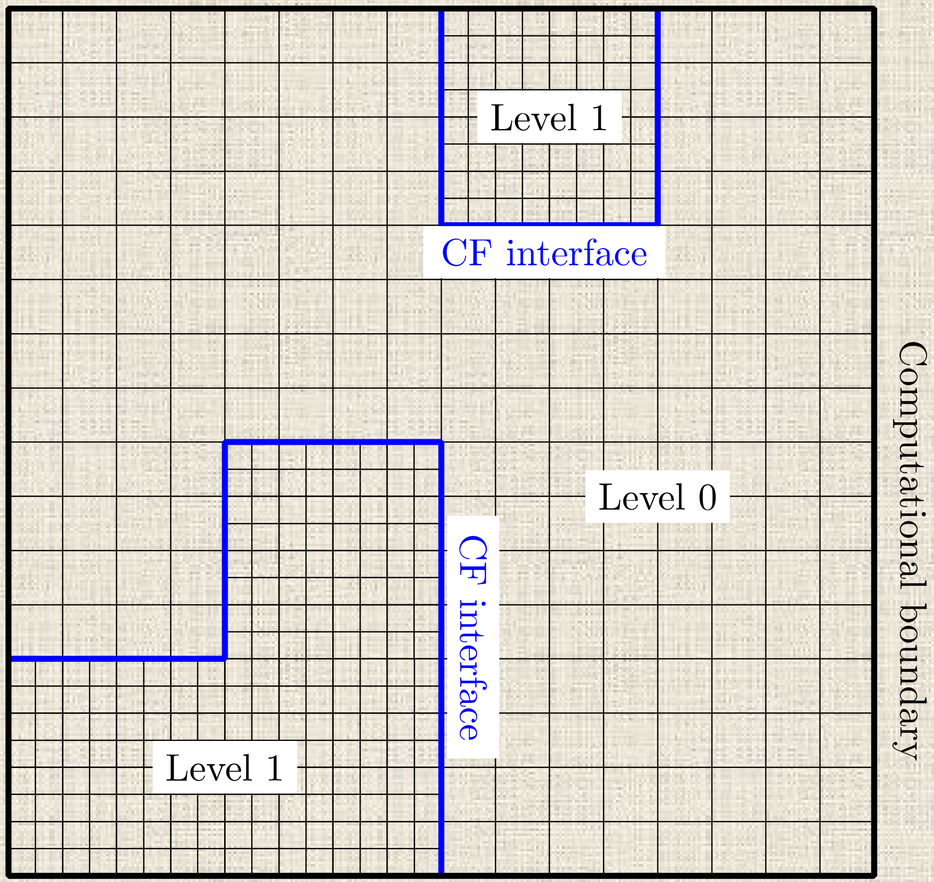
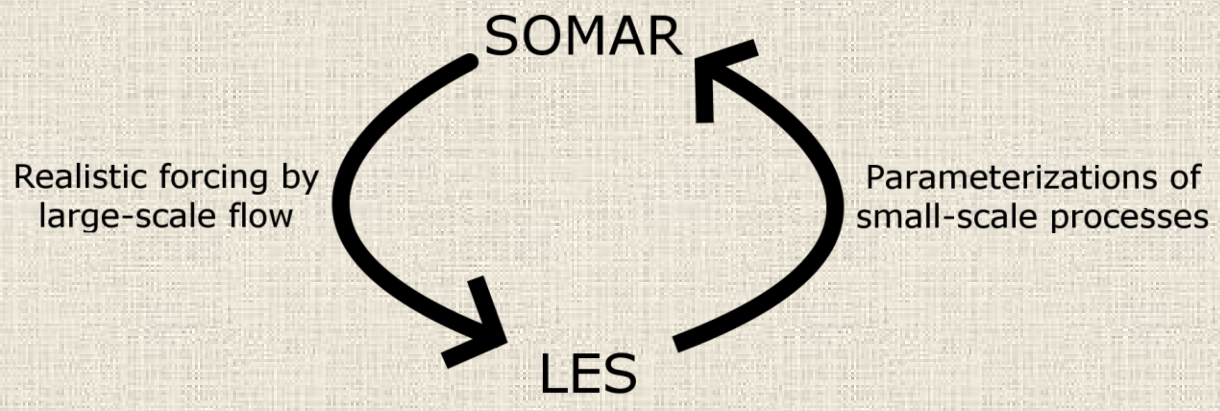
SOMAR-LES Algorithm





SOMAR-LES Equations

Coarse-grid equations

$$\nabla \cdot \mathbf{u} = 0,$$
$$\frac{D\mathbf{u}}{Dt} = \nu \nabla^2 \mathbf{u} - \nabla \cdot \boldsymbol{\tau} - -f\hat{\mathbf{k}} \times \mathbf{u} + b^* \hat{\mathbf{k}} - \nabla p^*$$
$$\frac{Db^*}{Dt} = \kappa \nabla^2 b^* - \nabla \cdot \boldsymbol{\lambda} + wN^2$$

Fine-grid equations

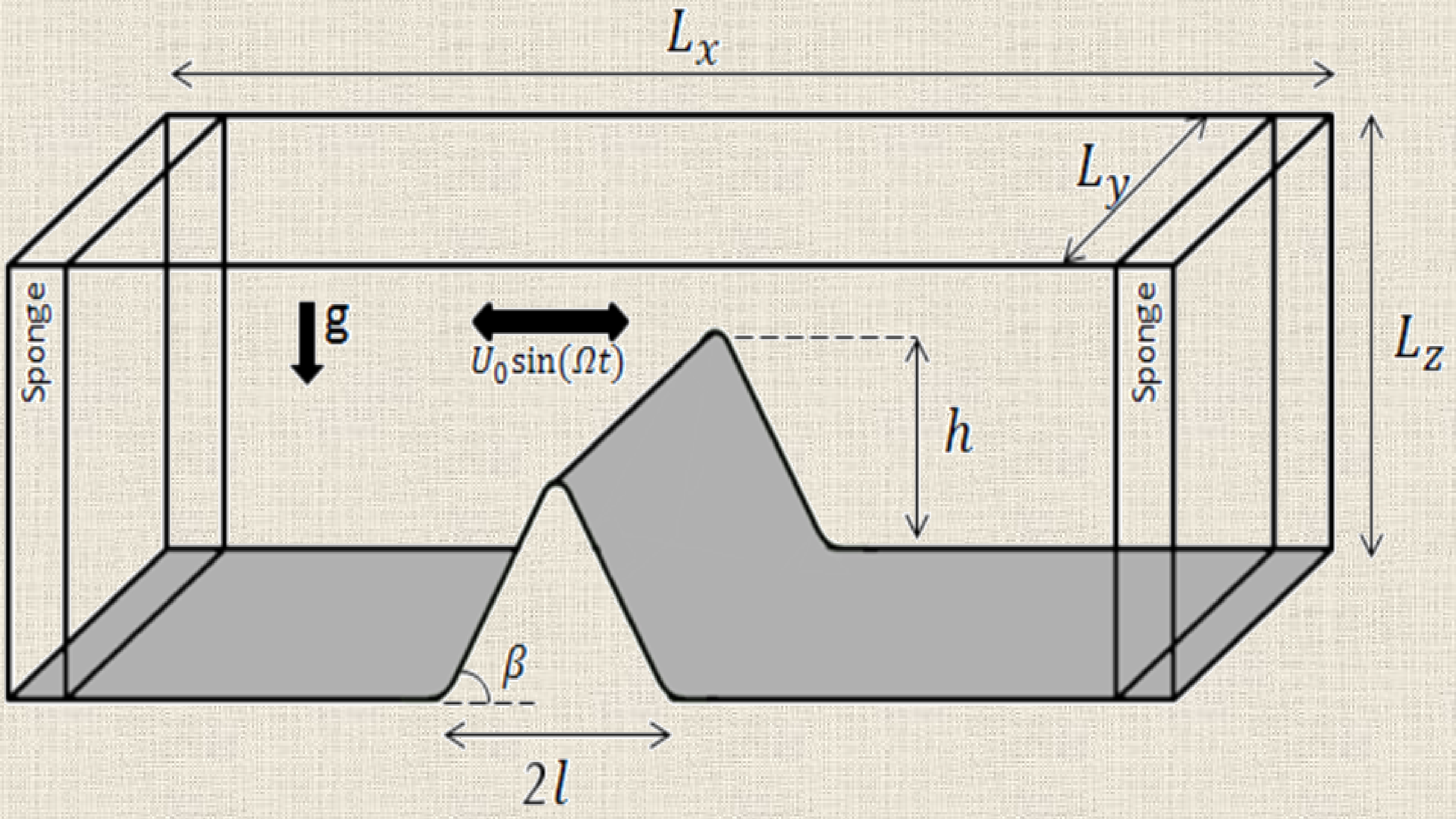
$$\nabla \cdot \overline{\mathbf{u}} = 0$$
$$\frac{D\overline{\mathbf{u}}}{Dt} = \nu \nabla^2 \overline{\mathbf{u}} - \nabla \cdot \boldsymbol{\tau} - f\hat{\mathbf{k}} \times \overline{\mathbf{u}} + \overline{b}^* \hat{\mathbf{k}} - \nabla p^*$$
$$\frac{D\overline{b}^*}{Dt} = \kappa \nabla^2 \overline{b}^* - \nabla \cdot \boldsymbol{\lambda} + \overline{w} N^2$$
$$\left\{ \begin{aligned} \boldsymbol{\tau} &= -2\nu_{sgs} \overline{\boldsymbol{S}} \\ \boldsymbol{\lambda} &= -\kappa_{sgs} \nabla \overline{b}^* \end{aligned} \right\}$$

$$\textbf{Smagorinsky} : \nu_{sgs} = (C_S \Delta)^2 \sqrt{2\overline{\boldsymbol{S}} : \overline{\boldsymbol{S}}}$$
$$\textbf{Ducros} : \nu_{sgs} = 0.0014\, C_K^{-3/2}\, \overline{\Delta}\, \sqrt{\overline{F_2}}$$
$$\begin{aligned} \overline{F_2} = \frac{1}{6} \Big(& ||\tilde{\overline{u}}_{i+1,j,k} - \tilde{\overline{u}}_{i,j,k}||^2 + ||\tilde{\overline{u}}_{i-1,j,k} - \tilde{\overline{u}}_{i,j,k}||^2 \\ & + ||\tilde{\overline{u}}_{i,j+1,k} - \tilde{\overline{u}}_{i,j,k}||^2 + ||\tilde{\overline{u}}_{i,j-1,k} - \tilde{\overline{u}}_{i,j,k}||^2 \\ & + ||\tilde{\overline{u}}_{i,j,k+1} - \tilde{\overline{u}}_{i,j,k}||^2 + ||\tilde{\overline{u}}_{i,j,k-1} - \tilde{\overline{u}}_{i,j,k}||^2 \Big) \end{aligned}$$

For more details: Chalamalla et al. (*Ocean Modelling*, 2017)

SOMAR-LES Simulations

Internal tide generation with AMR



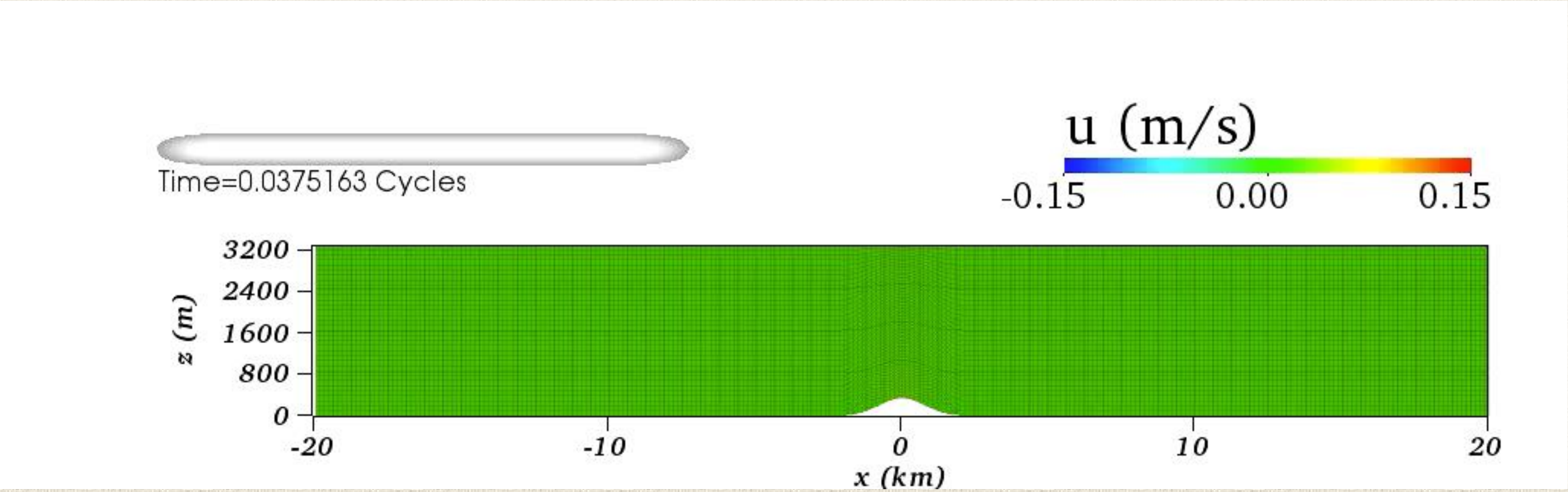
L_x = 40 km
L_y = 0.25 km
L_z = 3.28 km

dx = 9.8 m
dy = 3.9 m
dz = 3.2 m

$$Ex = \frac{U_0}{l\,\Omega} = 0.4$$

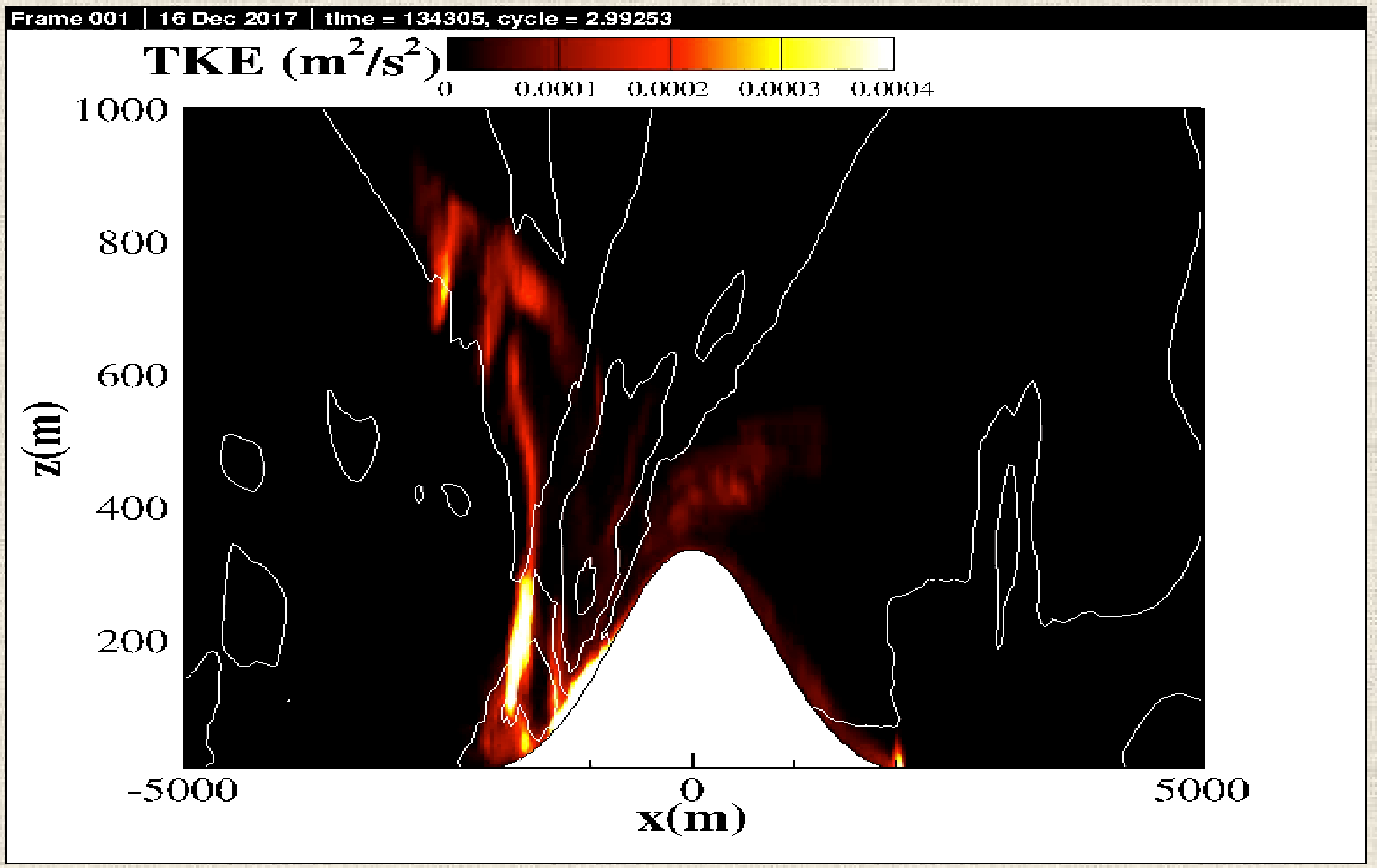
$$\epsilon = \frac{\tan(\beta)}{\tan(\theta)} \approx 1$$

Internal tide generation with AMR



- Refinement only in the localized regions near the bathymetry.
- Refinement is done based on the gradient Richardson number $Ri_g < 0.25$.

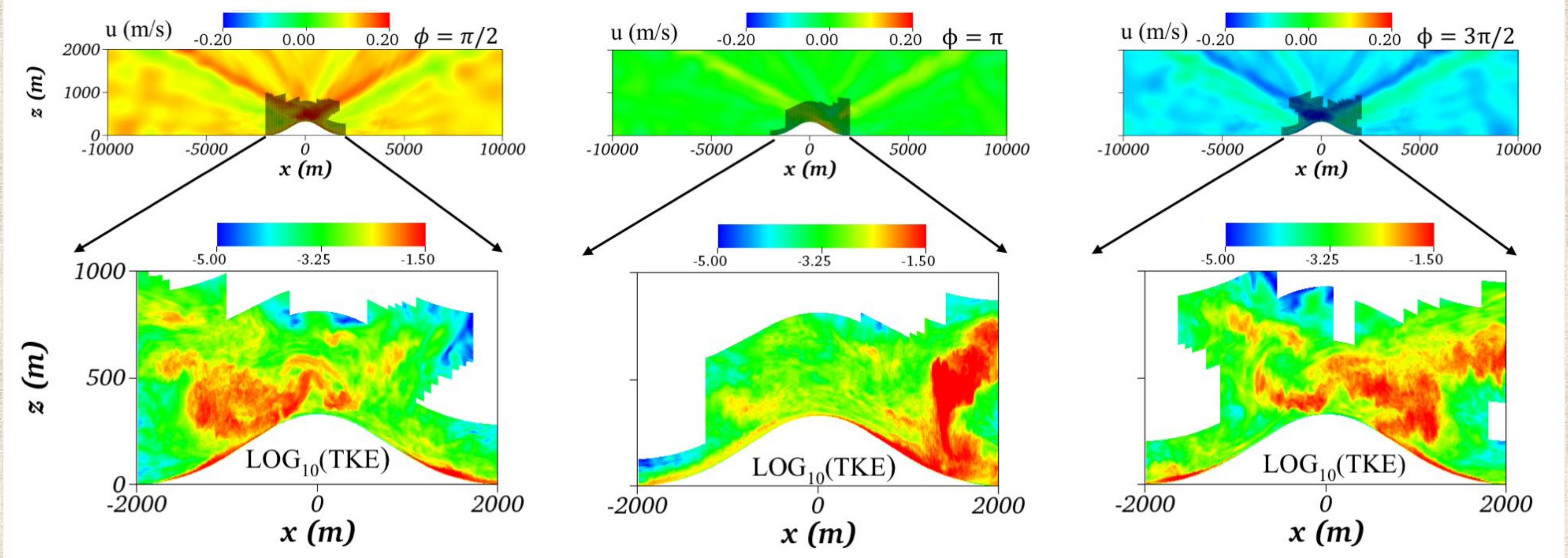
Internal tide generation with AMR



- Turbulence is intermittent in both space and time.

- Ideal problem for AMR, however the locations of turbulence needs to be predicted accurately.

Adaptive refinement + Subgrid scale model



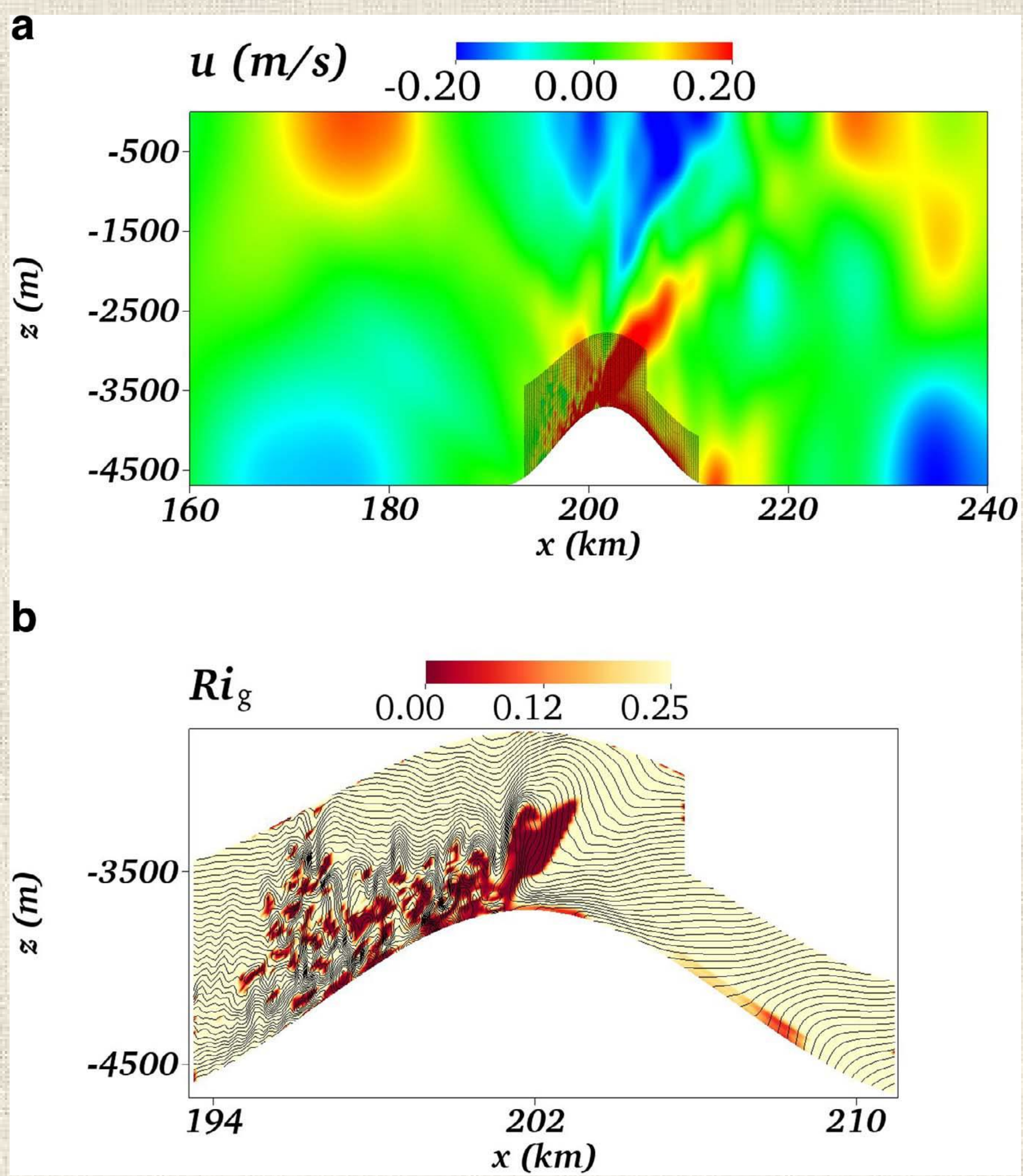
- Fine level grid adapts following the turbulent overturns.
- Turbulent overturns as tall as 500 m are found at certain phases.
- Need to a better job in predicting where the fine level grid is required.

Model validation

Model	C	M	q = 1-M/C	P
SOMAR-LES	0.736	0.637	0.13	0.036
LES	0.721	0.612	0.15	0.031

- Baroclinic energy budget and turbulent statistics compare well with previous numerical studies.
- Residual for baroclinic energy budget is less than 1%
- Fine level grid occupies less than 2% of the total computational domain.
- Total computational cost is just 10% of the single level grid solver.

Low mode wave scattering



L_x = 446 km dx = 109 m

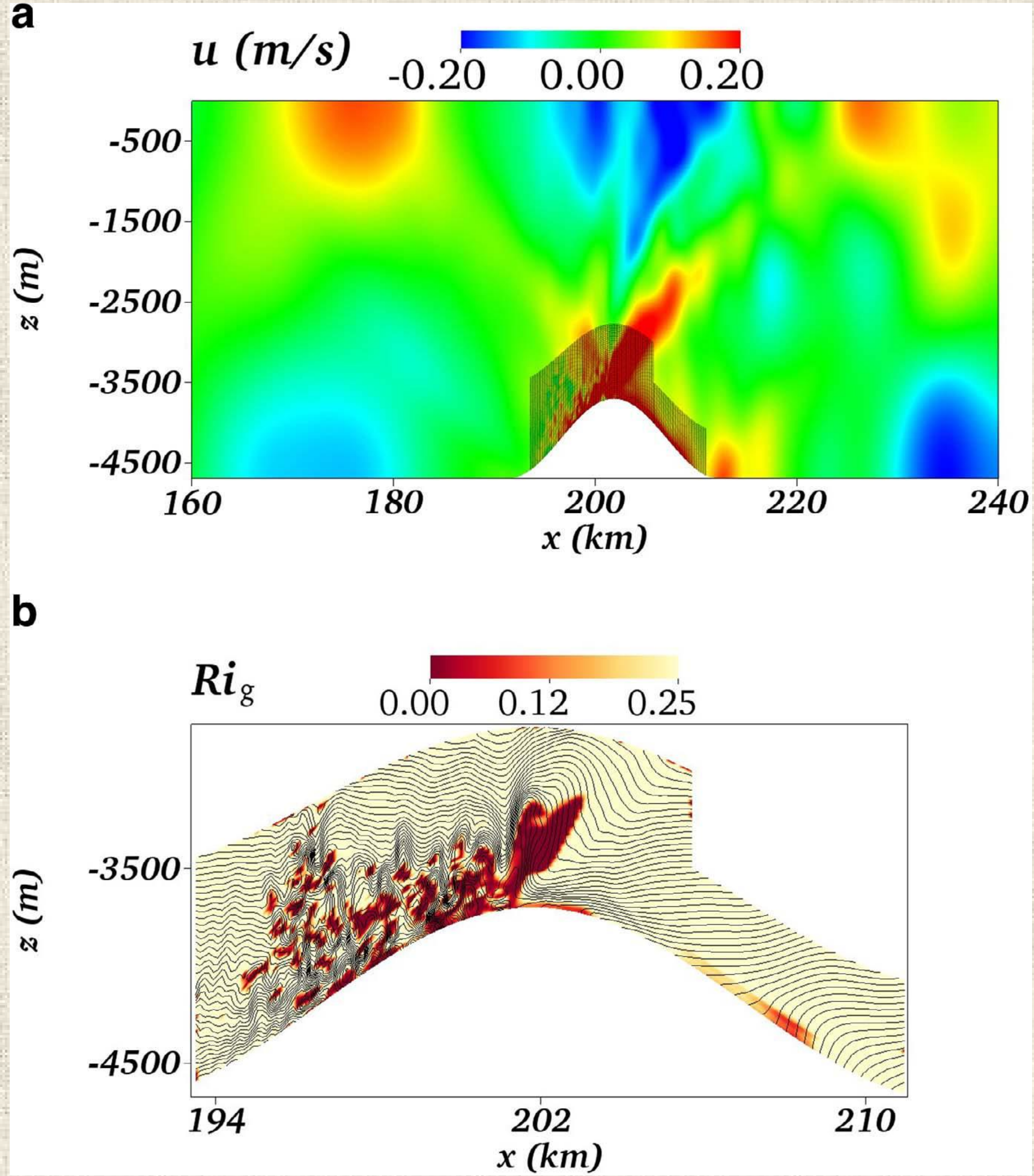
L_y = 127 km dy = 125 m

L_z = 4.7 km dz = 18 m

Topographic width = 21 km

- Interaction of low-mode internal wave with an isolated bathymetry results in the generation of higher modes due to nonlinear interaction with the sloping bottom.
- Fine grids exist where Gradient Richardson number is < 0.25.

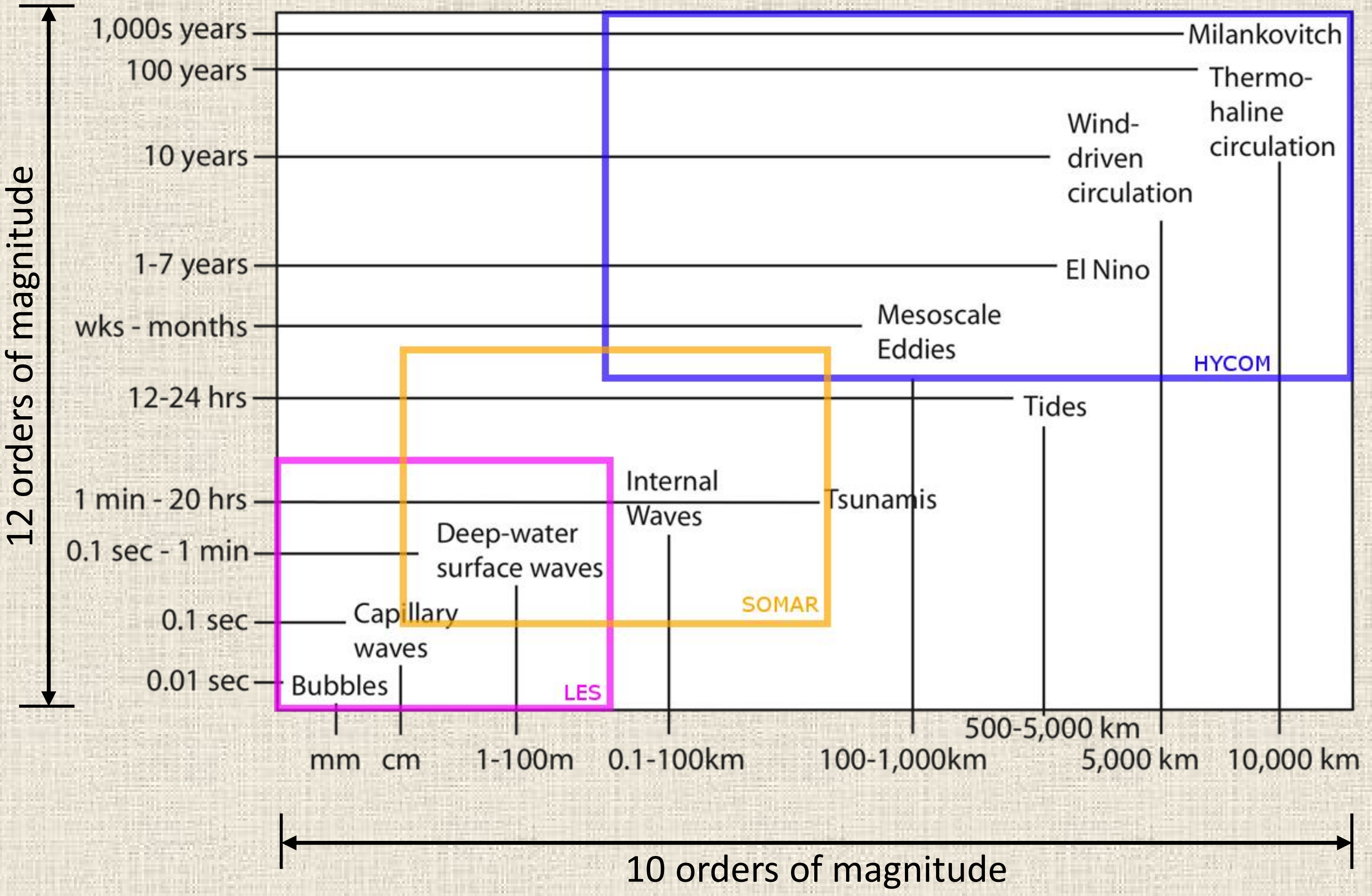
Low mode wave scattering



SOMAR-LES vs. MITgcm

- SOMAR-LES used 3x finer resolution than MITgcm.
- MITgcm used 4x more CPU-hours than SOMAR-LES.
- Reflected and transmitted energy in close agreement with linear theory.
- LES produces less dissipation than MITgcm's Thorpe-scale based eddy viscosity model.
- Work in progress: Add more levels of refinement + apply LES on more refined grid.

HYCOM-SOMAR-LES



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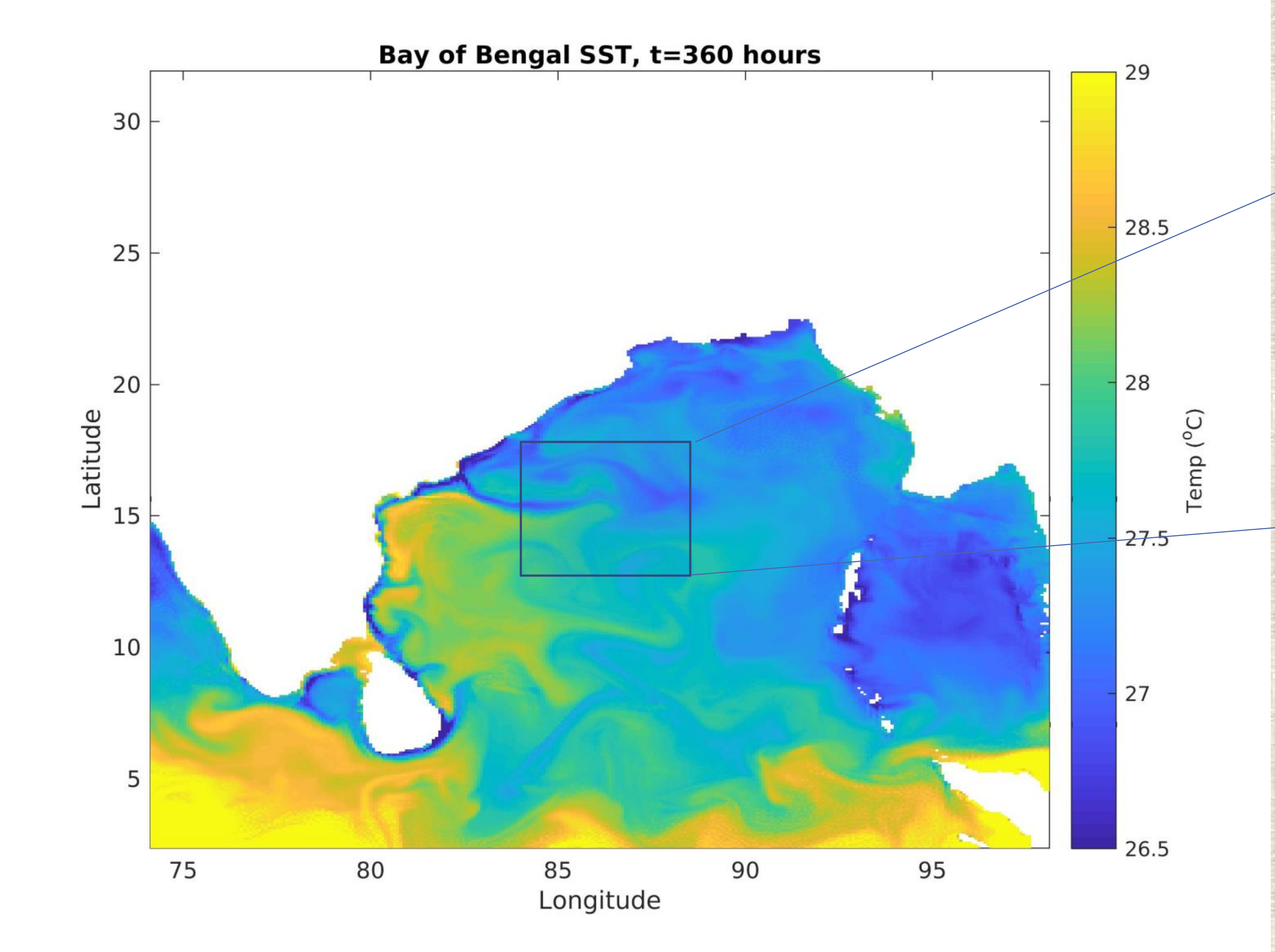
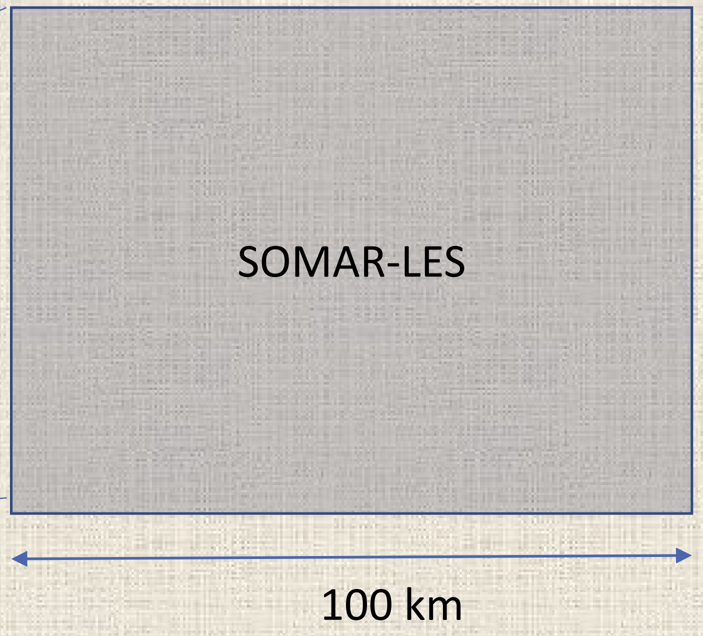
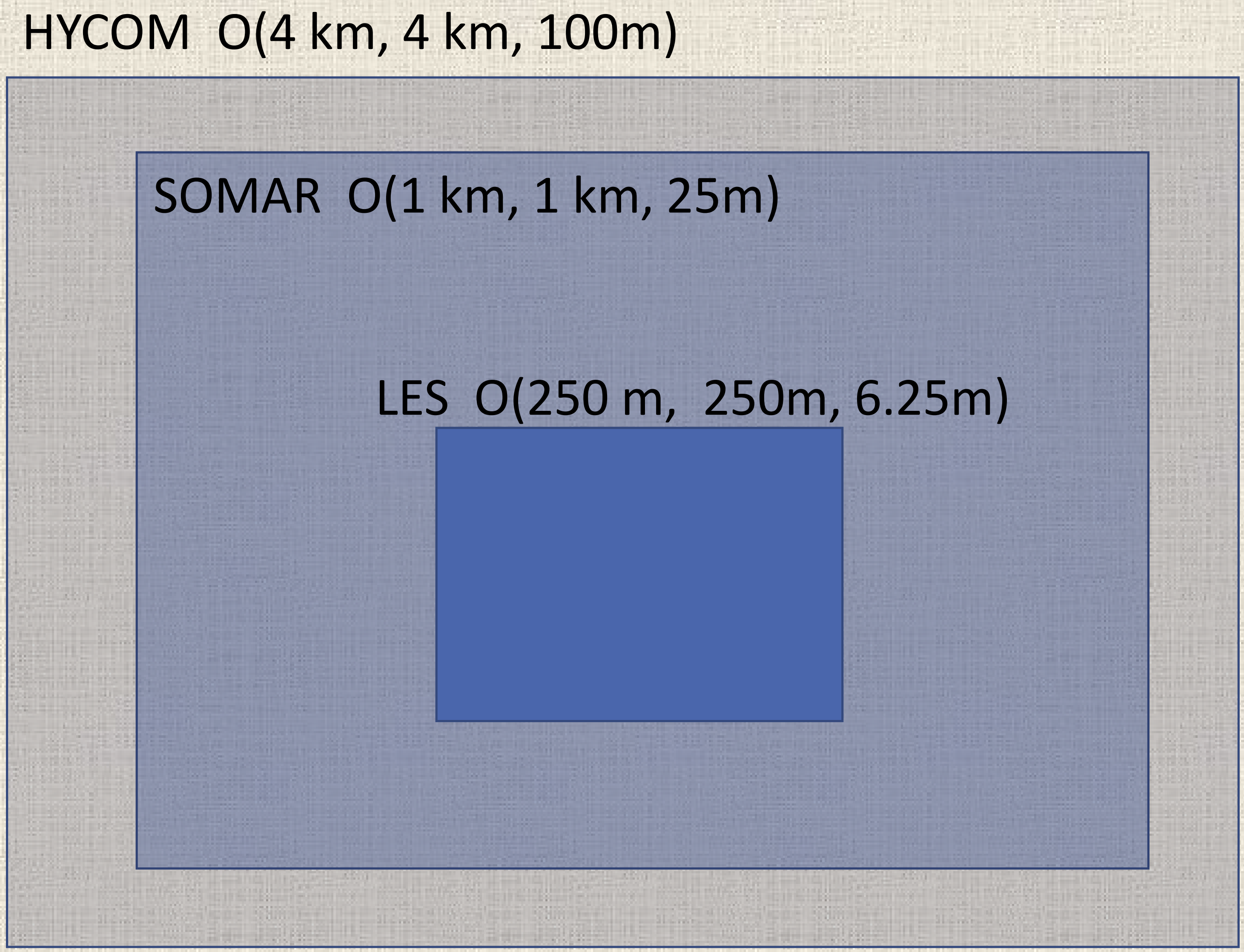
HYCOM-SOMAR-LES





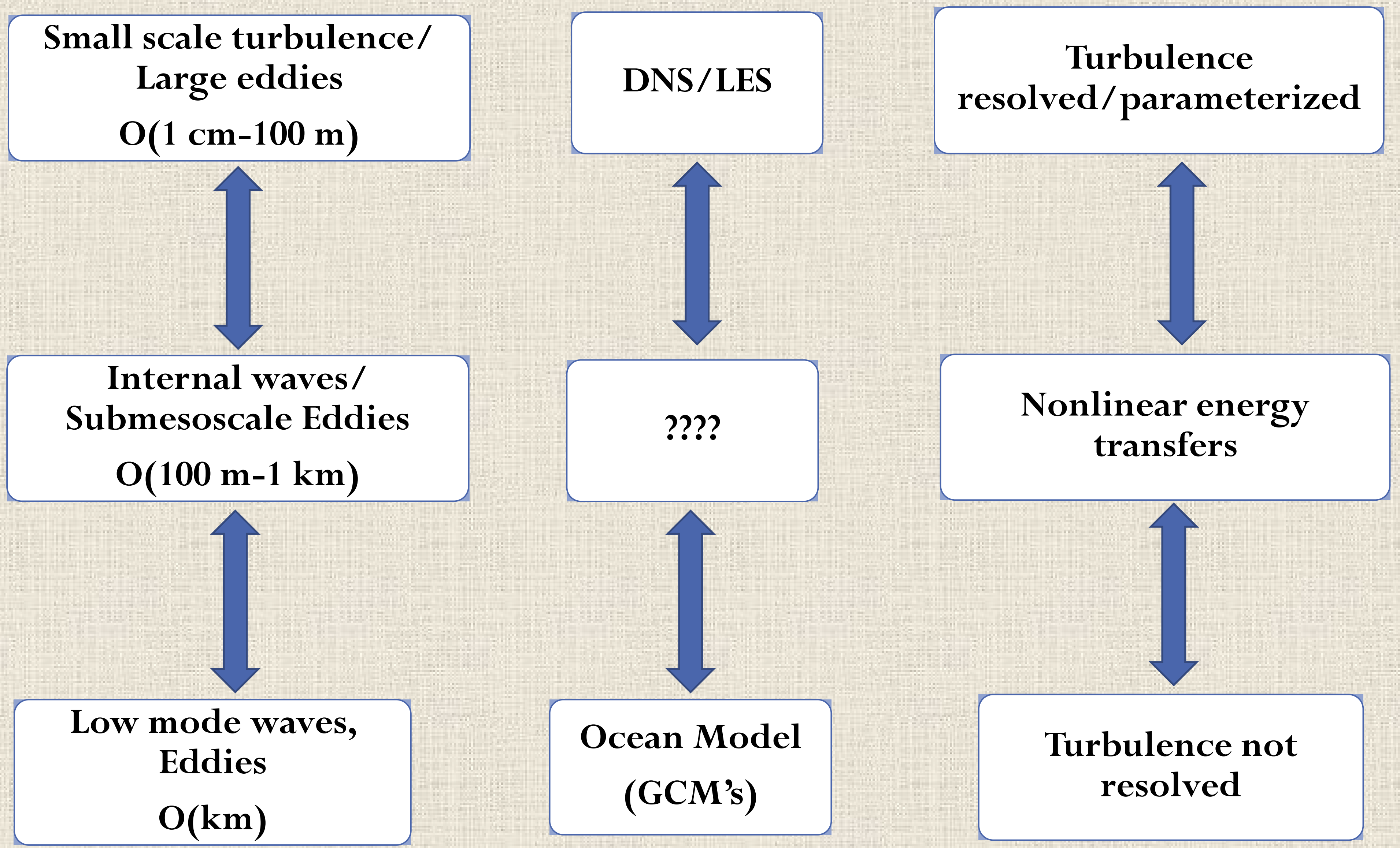
Future work with HYCOM-SOMAR-LES



- To study ocean-atmosphere interactions in the Bay of Bengal region by coupling this with atmospheric model.
- Achieve finer resolution, O(100m) in the horizontal and O(5m) in the vertical, to resolve the submesoscale eddies and their energy transfer to turbulence.
- With finer resolution, we hope to numerically model features which are observed in the field experiments and absent in the global ocean simulations.
- Address challenges that exist during exchange of information at interfacial boundaries.

Thank you!

Why multi-scale modeling ?



Need for a unified model !

Why multi-scale modeling ?

Small scale turbulence/
Large eddies

O(1 cm-100 m)

Internal waves/
Submesoscale Eddies

O(100 m-1 km)

DNS/LES

????

Turbulence
resolved/parameterized

To resolve nonlinear
energy transfers

Why multi-scale modeling ?

Small scale turbulence/ Large eddies O(1 cm-100 m)	DNS/LES	Turbulence resolved/parameterized
Internal waves/ Submesoscale Eddies O(100 m-1 km)	????	To resolve nonlinear energy transfers
Low mode waves, Eddies O(km)	Ocean Model (GCM's)	Turbulence not resolved

Large eddy simulations ..

- Very expensive to resolve all the scales of the flow.
- Smallest length scales are removed via low pass filtering of Navier-Stokes equations.
- Effect of the unresolved scales are modeled via sub-grid scale modeling.

$$\nabla . \bar{u} = 0$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}.\nabla \bar{u} = -\nabla p^* + \nu \nabla^2 \bar{u} - \frac{g}{\rho_0} \rho^* \hat{z} + \vec{F}_{ext} - \nabla . \tau$$

$$\frac{\partial \rho^*}{\partial t} + \bar{u}.\nabla \rho^* = \kappa \nabla^2 \rho^* - \bar{w} \frac{d\rho^b}{dz} - \nabla . \lambda$$

Eddy viscosity models

- Subgrid-scale stress tensor & density flux are defined as ..

$$\tau_{ij} = -2\, \nu_{sgs}\, S_{ij}$$

$$\lambda_j = -\kappa_{sgs} \frac{\partial b^*}{\partial x_j}$$

- Subgrid-scale viscosity & diffusivity is then computed using a variety of models.

- Smagorinsky model

$$\nu_{sgs}(\boldsymbol{x}, t) = (C_s \Delta^2)|S|$$

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Fine-grid equations

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$$\boldsymbol{\tau} = -2\nu_{sgs} \overline{\boldsymbol{S}}$$

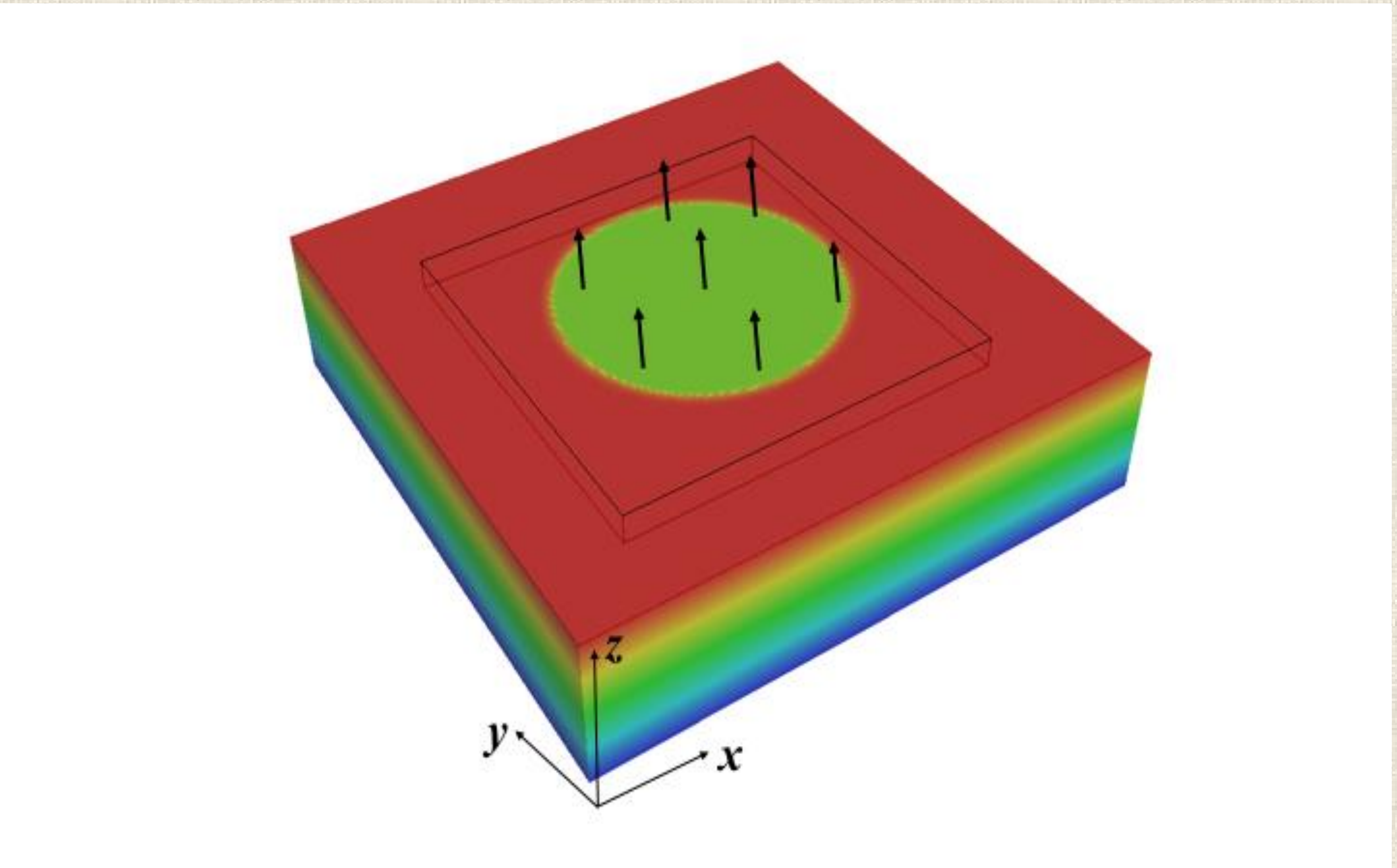
$$\boldsymbol{\lambda} = -\kappa_{sgs} \nabla \overline{b}^*$$

$$\nu_{sgs} = \left(C_S \sqrt[3]{\Delta x \, \Delta y \, \Delta z}\right)^2 \sqrt{2 \overline{\boldsymbol{S}} : \overline{\boldsymbol{S}}}$$

Add Ducros!

Chalamalla et al. (*Ocean Modelling*, 2017)

Schematic

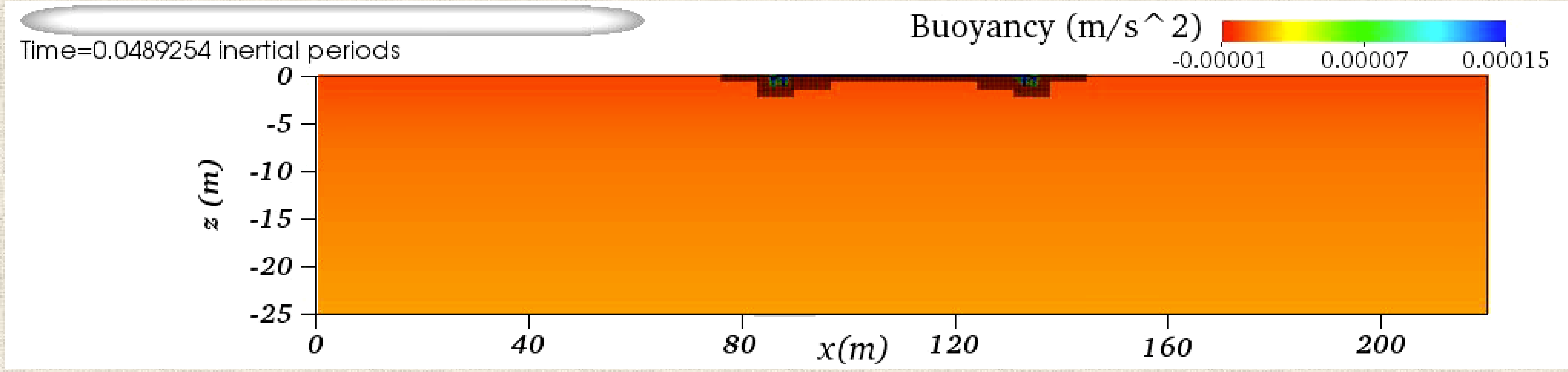


- Surface cooling is applied by adding forcing term to r.h.s of the density equation.
- Two-level grid with adaptive refinement is used to dynamically resolve convective plumes.

$$Rayleigh\ number = \frac{B_0 H^4}{\kappa^2 \nu}$$

$$Rossby\ number = \frac{B_0^{1/2}}{f^{3/2} H}$$

SOMAR-LES Animations



- Localized refinement only in the regions of turbulence based on gradient Richardson number criteria $Ri_g < 0$

- Add some points .. About efficiency of SOMAR in these kind of problems ..